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Comparison of Reciprocity and Closure Enforcement Methods for Radiation View Factors

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The view factors that are used in diffuse-gray radiation enclosure calculations are often computed by approximate numerical integrations. These approximately calculated view factors will usually not satisfy the important physical constraints of reciprocity and closure. In this article several view-factor rectification algorithms are reviewed. A Monte Carlo investigation is undertaken to study the propagation of view-factor uncertainties into the heat transfer results of the diffuse-gray enclosure calculations. It is found that the weighted least-squares algorithm is vastly superior to the other rectification schemes for the reduction of heat flux uncertainties resulting from uncertainties in view factors. In a sample problem, which has proven to be very sensitive to uncertainties in view factor, the heat transfer calculations with weighted least-squares rectified view factors are very good with an original view-factor matrix computed to only one-digit accuracy.

Nomenclature

a_k = area of surface k
b = vector of coefficients in Eq. (3)
c = vector of coefficients in Eq. (16)
D_a = diagonal matrix of areas
D_e^M = diagonal matrix with zeros for elements 1 to M and ε_k for elements $k = M + 1$ to N
D_e^{*M} = diagonal matrix with ε_k for elements $k = 1$ to M and zeros for elements $M + 1$ to N
d_i = correction factor in Leersum's scheme
F = matrix of view factors
f = vector of view factors
f_{ij} = element i, j of F
f_{Lij} = Leersum's rectified view factors
f_{Nij} = naive rectified view factors
f_{null} = null-space component of view-factor vector, Eq. (19)
f_{oij} = nonlinear programming solution values
f_{opt} = least-squares projection values, Eq. (20)
f_{row} = row-space component of view-factor vector, Eq. (16)
I = identity matrix
i, j, k = used as indexes
m = number of nonzero values used in Leersum's scheme
N = total number of surfaces
N_b = null-space bias matrix
q_k = net heat flux on surface k
$q_{0,k}$ = radiosity leaving surface k
q_0 = vector of radiosities
R = closure and reciprocity matrix, Eq. (14)
r = intermediate result vector
S = standard deviations matrix corresponding to F
t_k = temperature on surface k
V = covariance matrix
w_{ij} = weights used in Eq. (10)
y = weighed sum-square error, Eq. (10)
ε_k = emissivity on surface k
σ = Stefan–Boltzmann constant

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Introduction

IT is general knowledge in the radiation heat transfer literature that the view factors in diffuse-gray radiation enclosure calculations should be computed in such a way that they satisfy the physical constraints of reciprocity and closure. For systems with a large number of surfaces, the only practical way to compute the view factors is by approximate numerical integrations. Monte Carlo integration is a popular technique that is robust and has the added advantage of providing an estimate of the uncertainty in each calculation. These approximately computed view factors will often not satisfy the reciprocity and closure constraints, and artificial means of enforcement must be adopted.

This article is concerned with the fidelity of the heat transfer calculations, not the fidelity of the view factors. If the view factors can be computed with high precision, of course no rectification algorithm is required. As stated, it is not always practical to compute the view factors with this precision. Also, it may not be efficient to compute the view factors to such high precision. As discussed by Taylor et al.,[1] beyond a certain view-factor precision the uncertainty in the heat transfer calculations is controlled by the uncertainty in radiation properties, geometric dimensions, and process specifications such as surface temperatures. Taylor et al. present a detailed sensitivity analysis that can be used to judge the interplay of these various uncertainty sources.

Most heat transfer textbooks adopt a naive enforcement in which 1) only the view factors above the diagonal in the view-factor matrix are computed, 2) the view factors below the diagonal are computed using reciprocity relationships, and 3) the view factors along the diagonal are computed using closure. Sowell and O'Brien[2] present a refinement of the naive enforcement that computes a full set of view factors from any independent set. Tsuyuki[3] presents a refined form of the naive enforcement that avoids negative view factors. Van Leersum[4] presents an iterative approach that enforces closure and reciprocity on an approximate set of view factors and avoids negative instances. Vercammen and Froment[5] and Larsen and Howell[6] present rectification algorithms based on least-squares regression that were originally applied to gas-radiation exchange areas, but which can be readily adapted to view-factor rectification.

Taylor et al.[1,7] have demonstrated that diffuse-gray radiation enclosure problems can be very sensitive to errors in the view factors. In their work, they found that the simultaneous enforcement of reciprocity and closure using the naive algorithm described previously will greatly reduce this sensitivity. This is directly contrary to the conclusions drawn by Murty and Murty[8] who concluded, based on a limited set of case studies of zone-radiation calculations, that the rectification procedure and, in fact, rectification itself, had no effect on the results of the calculations.

This article extends the previous work of Taylor et al. by considering more advanced reciprocity and closure enforcement algorithms and comparing the propagation of the view-factor errors into computed heat flux results of the diffuse-gray enclosure analysis for the different methods.

Four view-factor rectification algorithms are discussed and compared: 1) no enforcement—all view factors independently computed with neither reciprocity nor closure enforced, 2) naive enforcement, 3) Van Leersum's enforcement, and 4) least-squares optimal enforcement.

The technique used for comparison is a Monte Carlo uncertainty analysis of a sample problem that has proven to be hypersensitive to errors in the view factors when reciprocity and closure are not enforced. The result is the distribution in computed surface heat flux for assumed uncertainty distributions of the original unrectified view factors.

Diffuse-Gray Enclosure Formulation

Radiation exchange between finite diffuse-gray areas that form an enclosure is discussed in almost all general heat transfer textbooks. Excellent detailed discussions can be found in any thermal radiation heat transfer textbook (e.g., Brewster[9] and Siegel and Howell[10]). The basic restrictions are that each surface have uniform temperature, uniform radiative properties that are diffuse and gray, and uniform radiosity. Boundary conditions for the kth surface are expressed by specifying either q_k or t_k. Mixed boundary conditions cause no problem. If all of the surfaces with specified heat flux are considered first as surfaces 1 through M and the surfaces with specified temperatures numbered $M + 1$ through N, the following set of linear equations can be obtained for the radiosity values[10]:

$$q_{0,k} - \frac{1}{a_k}\sum_{j=1}^{N} a_j q_{0,j} f_{jk} = q_k, \qquad k = 1, \ldots, M \tag{1}$$

$$q_{0,k} - (1 - \varepsilon_k)\frac{1}{a_k}\sum_{j=1}^{N} a_j q_{0,j} f_{jk} = \varepsilon_k \sigma t_k^4$$
$$k = M + 1, \ldots, N \tag{2}$$

These equations are more conveniently expressed in matrix form as

$$[I - (I - D_e^M)D_a^{-1}F^T D_a]\boldsymbol{q}_0 = \boldsymbol{b} \tag{3}$$

where the terms are defined in the Nomenclature.

After solving Eq. (3) for the radiosities, Eqs. (2) are then applied to surfaces $k = 1$ to M to compute the temperatures of the surfaces with specified heat fluxes, and Eqs. (1) are used to compute the heat fluxes on the surfaces with specified temperatures. If the result $\boldsymbol{r}$ is taken to be the vector whose first M elements are $\varepsilon_k \sigma t_k^4$ and whose last N-M elements are q_k, the final equation is

$$\boldsymbol{r} = [I - (I - D_e^{*M})D_a^{-1}F^T D_a]\boldsymbol{q}_0 \tag{4}$$

where D_e^{*M} is the complement of D_e^M.

Usually at this stage of development, the view-factor reciprocity relationship is substituted into Eqs. (3) and (4) to simplify the equations. However, in this investigation, we are interested in cases where reciprocity is not strictly enforced. In that case, it is more appropriate to work with Eqs. (3) and (4).

View-Factor Rectification

Three view-factor rectification schemes are considered: 1) naive, 2) Leersum's, and 3) least-squares optimum. For the least-squares optimum three subsets are considered: 1) unweighted without nonnegativity, 2) unweighted with nonnegativity, and 3) weighted with nonnegativity. Each of these procedures is discussed next.

Naive Rectification

For the naive rectification the view factors are considered to form a matrix F. Only the view factors above the main diagonal are considered and all others are discarded. The view factors below the main diagonal are computed from reciprocity

$$a_i f_{Nij} = a_j f_{Nji} \tag{5}$$

and the diagonal elements are computed using the closure relation

$$f_{Nii} = 1 - \sum_{\substack{j=1 \\ j \neq i}}^{N} f_{Nij} \tag{6}$$

No attempt is made to ensure nonnegative view factors. The physically impossible negative view factors are naively accepted.

Leersum's Rectification

Van Leersum[4] has published an iterative scheme that can be considered a refinement of the naive rectification. His method spreads the closure adjustments over all of the view factors and assures nonnegative view factors. His algorithm is given next.

1) For each row in F, compute a correction factor based on closure

$$d_i = \left(1 - \sum_{k=1}^{N} f_{ik}\right) \Big/ m \tag{7}$$

where m is the number of nonzero view factors in row i.

2) For each nonzero view factor in row i apply the correction

$$f_{Lik} = f_{ik} + d_i, \qquad k = 1, \ldots, N \tag{8}$$

If any $f_{Lij} < 0$, decrease m by the number of negative values and recalculate d_i bypassing the view factors that made the previous f_{Lij} negative. Repeat this procedure until no negative view factors are obtained.

3) Enforce reciprocity by computing values for column i

$$f_{Lki} = a_i f_{Lik}/a_k, \qquad k = 1, \ldots, N \tag{9}$$

Steps 2 and 3 are carried out for each row–column pair.

4) Repeat this process in turn for each row.

5) Since the enforcement of reciprocity in 3 disturbs the closure forced in 1 and 2, the entire process is repeated iteratively until the values of d_i are arbitrarily small. This process always converged for the cases considered herein.

The step-by-step enforcement of reciprocity in 3 overwrites all of the original view factors below the main diagonal; therefore, Leersum's procedure only considers the diagonal and upper triangular elements in the original view-factor matrix.

Least-Squares Optimum

The least-squares optimization problem can be posed as the nonlinear programming problem to minimize the weighted error

$$y = \sum_{i=1}^{N}\sum_{j=1}^{N} w_{ij}(f_{oij} - f_{ij})^2 \tag{10}$$

where the f_{ij} are the original approximately determined view factors, f_{oij} are the corrected view factors, and w_{ij} are the weights used when the view factors have unequal uncertainty. The closure and reciprocity constraints are

$$\sum_{j=1}^{N} f_{oij} = 1, \qquad i = l, \ldots, N \tag{11}$$

$$a_j f_{oji} - a_i f_{oij} = 0$$
$$i = 1, \ldots, N-1, \quad j = i+1, \ldots, N \tag{12}$$

If nonnegativity is desired, the following inequality constraints can be applied:

$$f_{oij} \geq 0 \qquad i = 1, \ldots, N \quad j = 1, \ldots, N \tag{13}$$

This problem can be readily solved using any number of nonlinear-programming techniques. This technique is for all practical purposes, the same as those of Vercammen and Froment[5] and Larsen and Howell.[6]

However, considerable insight can be gained and a computational formula can be derived if the problem is viewed from a geometric standpoint. First, the view factors are grouped into a column vector instead of a matrix. The view-factor matrix is stacked in row-major form. Closure and reciprocity are enforced by applying the equality constraints [Eqs. (11) and (12)] to form a set of linear equations

$$R \cdot f = c \tag{14}$$

The 2 × 2 system would yield, e.g.,

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & a_1 & -a_2 & 0 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{21} \\ f_{22} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \tag{15}$$

Equations (14) have $N(N-1)/2$ degrees of freedom. The naive rectification is obtained by specifying the $N(N-1)/2$ view factors above the main diagonal and computing the remainder from Eqs. (14). However, it is more desirable to use all of the N^2 view factors.

The solutions of Eqs. (14) can be factored into two orthogonal subspaces, the row-space (particular solutions) and the null-space (homogeneous solutions). The row-space component of the solution is computed using the expression[11]

$$f_{\text{row}} = R^T(RR^T)^{-1}c \tag{16}$$

This vector is the particular solution of Eqs. (14), which has the least norm, and it is a necessary component of all solutions of Eqs. (14). The other component of the solution $(f - f_{\text{row}})$ should lie in the null-space of R and can be expressed as a linear combination of basis vectors for the null-space

$$(f - f_{\text{row}}) = N_b x \tag{17}$$

where N_b is the matrix whose columns form that basis and x contains the weights of the linear combination. However, if there are errors in the computed view factors f, Eqs. (17) will not be consistent, and we must resort to the least-squares solution[11]:

$$x = (N_b^T N_b)^{-1} N_b^T (f - f_{\text{row}}) \tag{18}$$

The projection of $(f - f_{\text{row}})$ onto the null-space, then, is the desired set of corrected view factors

$$f_{\text{null}} = N_b(N_b^T N_b)^{-1} N_b^T (f - f_{\text{row}}) \tag{19}$$

and the least-squares optimum set of view factors is

$$f_{\text{opt}} = f_{\text{row}} + f_{\text{null}} \tag{20}$$

When the data are not all equally reliable (usually the case for view factors), weighted least squares should be used for the solution of Eqs. (17) (Ref. 11)

$$f_{\text{null}}^{w} = N_b(N_b^T V^{-1} N_b)^{-1} N_b^T V^{-1}(f - f_{\text{row}}) \tag{21}$$

The view-factor variances that serve as the weights are a direct result of Monte Carlo computations. For other methods such as numerical evaluation of double integrals or contour integrals, bounds on the numerical errors can be estimated. No other weighting functions are considered in this work. The weighted optimum view factors are then computed using

$$f_{\text{opt}}^{w} = f_{\text{row}} + f_{\text{null}}^{w} \tag{22}$$

The view-factor rectifications computed using Eqs. (20) and (22) do not enforce nonnegativity. The least-squares optimum view-factor rectification obtained through Eqs. (20) and (22) are exactly the same as those that would be obtained by solving the nonlinear programming problem in Eqs. (10–12) without considering the nonnegativity constraints.

Equations (16–22) perhaps look complicated to those who have not studied linear analysis in some depth. However, they represent a simple geometric concept. For example, consider a hypothetical static force balance where from physical constraints it is known that all valid forces must be in the x-y plane. If an independent approximate solution produces a force that does not lie in the x-y plane, the best use of that solution is to project onto the x-y plane, thus, throwing away the z component.

As discussed before, the view factors must be nonnegative to be physically realistic; a negative view factor is meaningless. It is our opinion and experience that allowing slightly negative values in the rectified view-factor matrix does not seriously impact the fidelity of the heat transfer results. Certainly, the strict enforcement of reciprocity and closure has had a much stronger impact on our results.

A two-step procedure that is easy to implement and closely approximates the results of the nonlinear-programming solution with the nonnegativity constraints is to apply Eqs. (20) or (22) and to assume that the equality in Eqs. (13) would be enforced on all negative values. These view factors are set to zero and removed from consideration obtaining a reduced-order problem, and the process is then repeated with the reduced set of data. This procedure has proven to give exactly the same set of rectified view factors as the nonlinear-programming solution in about 90% of the cases and only slightly different ones in the other 10% of the cases.

All of the computations that are given next in the numerical examples section were performed in Matlab®. For the small problem considered here, the computations were for all practical purposes instantaneous. For larger systems, the computations can be more efficiently computed using decomposition or iterative techniques instead of matrix inverses.

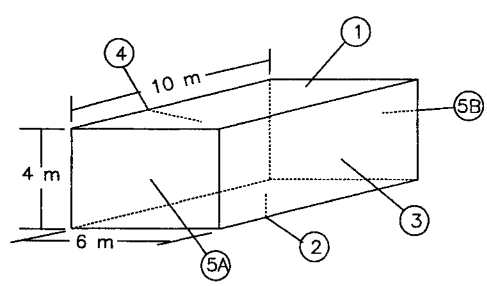

Fig. 1 Schematic of a room for the example problem.

Numerical Examples

The following problem from the heat transfer text by Incropera and Dewitt[12] is used as a basis of comparison of the different techniques in this article.

> 13.62 A room (Figure 1) is represented by the following enclosure, where the ceiling (1) has an emissivity of 0.8 and is maintained at 40°C by embedded electrical heating elements. Heaters are also used to maintain the floor (2) of emissivity 0.9 at 50°C. The right wall (3) of emissivity 0.7 reaches a temperature of 15°C on a cold, winter day. The left wall (4) and end walls (5A, 5B) are very well insulated. To simplify the analysis, treat the two end walls as a single surface (5). Assuming the surfaces are diffuse-gray, find the net radiation heat transfer from each surface.

This problem was the genesis of our interest in the subject of view-factor sensitivity and rectification. This problem was assigned in the second heat transfer course at Mississippi State University during the Fall 1992 term. Two students, Miguel and Simon, ignored the simplification and worked the problem as a six-sided enclosure. Miguel computed his view factors to four-digit accuracy and Simon to two-digit accuracy; they got radically different answers for the heat fluxes. An analysis of this problem and the cause for this hypersensitivity are discussed in a previous publication.[1]

In the following, only the effects of rectification on the computed heat fluxes are presented. The sensitivities of the computed surface temperatures to errors in the input parameters are less than those of heat fluxes because of the one-fourth power in Eq. (2). Taylor et al.[7] discuss the sensitivity of surface temperature to errors in view factor in a different context.

This problem is sufficient to demonstrate a comparison of the effects of the rectification algorithms on the fidelity of the heat transfer calculations. However, it is a very simple problem and does not significantly stress the rectification algorithms. The authors, in cooperation with Tsuyuki,[3] are currently preparing a larger scale problem to test the algorithms.

The view-factor matrix computed to four-digit accuracy is

$$F = \begin{bmatrix} 0.0 & 0.394 & 0.1921 & 0.1921 & 0.1109 & 0.1109 \\ 0.394 & 0.0 & 0.1921 & 0.1921 & 0.1109 & 0.1109 \\ 0.2881 & 0.2881 & 0.0 & 0.196 & 0.1139 & 0.1139 \\ 0.2881 & 0.2881 & 0.196 & 0.0 & 0.1139 & 0.1139 \\ 0.2774 & 0.2774 & 0.1898 & 0.1898 & 0.0 & 0.066 \\ 0.2774 & 0.2774 & 0.1898 & 0.1898 & 0.066 & 0.0 \end{bmatrix} \tag{23}$$

Seven different numerical experiments have been performed. In each case, the starting point was the view-factor matrix listed in Eq. (23). Random errors were then introduced by sampling a random number generator that produced normally distributed values. The difference in each experiment resulted from the way that the variance of these random errors was assigned to the view factors. A thousand trials were conducted for each case. In all of the following, covariance terms are assumed to be negligible.

Equal Variance

The first numerical experiment considers the view factors to have equal variance with a view-factor standard deviation of 0.01. Table 1 gives mean values of the heat flux for several of the rectification schemes. Since the variances are all equal and the covariances are assumed to be zero, the weighted least-squares optimum scheme would not give results different from the unweighted schemes and was not used. The base solution is computed using an unweighted least-squares rectification of the view factors in Eq. (23). Table 2 shows the standard deviations for the heat-flux calculations. From Table 1, all of the rectification schemes seem to have means that are roughly equal to the base solution. Table 2 shows, however, that there is a large difference in the standard deviations of the calculated heat fluxes. For surface 1, the case of no rectification has a standard deviation that is almost an order of magnitude larger than the rectified values. Figure 2 shows histograms of surface-3 heat-flux distributions for each rectification scheme and for no rectification. Surface 3 was chosen as a representative surface.

The tables and figure reveal that all of the rectifications are effective for this problem. The nonnegative least-squares procedures are about twice as effective in reducing errors in the heat-flux calculations as Leersum's rectification, which in turn is about twice as effective as the naive rectification. Among the least squares, the nonnegative projection scheme and the nonlinear-programming scheme yield almost identical results as expected, and the least squares without nonnegativity has very slightly larger errors in heat flux than its nonnegative counterparts.

The following cases consider unequal variances. Depending on the location of the errors in the view-factor matrix, the relative success of the rectification schemes is vastly different from that seen for equal variances.

Diagonal-Dominated Variances

For this case study the view factors along the main diagonal are considered to have standard deviations that are 100 times as large as the off-diagonal view factors.

Table 1 Mean heat-flux values for equal variance case

	Least squares						
Surface	Not nonnegative	Nonlinear programming nonnegative	Nonnegative	Leersum	Naive	No rectification	Base
1	−3.6273	−3.6030	−3.6036	−3.6231	−3.6570	−3.9179	−3.6266
2	83.9532	83.7224	83.7249	83.7249	83.9425	83.9205	83.9447
3	−120.4888	−120.1792	−120.1820	−120.4798	−120.3951	−120.3969	−120.4772

Table 2 Standard deviations in heat flux for equal variance case

	Least squares					
Surface	Not nonnegative	Nonlinear programming nonnegative	Nonnegative	Leersum	Naive	No rectification
1	0.4374	0.3449	0.3447	0.6364	1.0005	10.2714
2	0.5776	0.4394	0.4398	0.8416	1.5774	11.8809
3	0.7272	0.5138	0.5148	0.9571	2.2777	12.3575

Table 3 Mean heat-flux values for the diagonal-dominated variance case

Surface	Nonnegative least squares Unweighted	Nonnegative least squares Weighted	Leersum	Naive	Base
1	−3.4722	−3.6263	−3.4719	−3.6264	−3.6266
2	82.1136	83.8987	83.9797	83.9417	83.9447
3	−117.9622	−120.4087	−120.7616	−120.4729	−120.4772

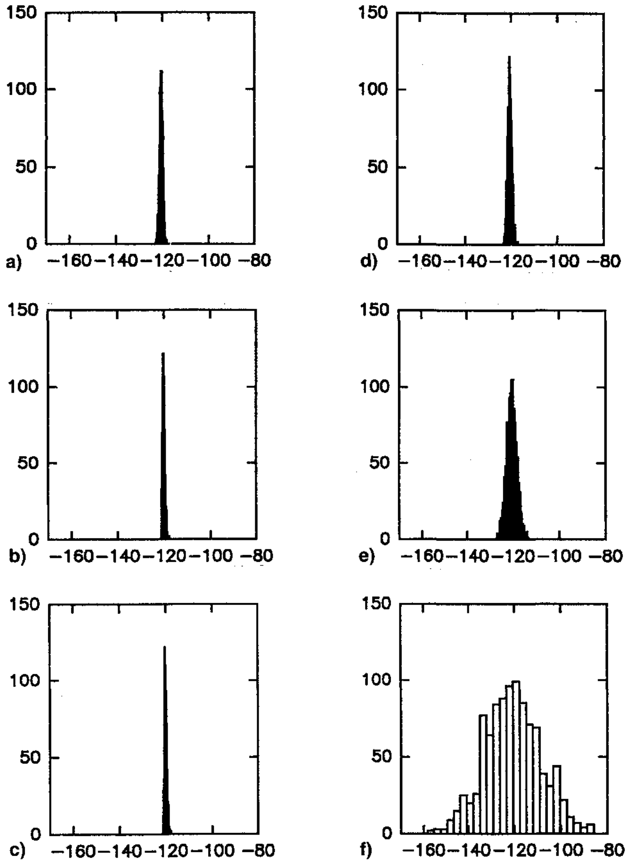

Fig. 2 Histogram of surface-3 heat flux (W/m²) for equal variance case: a) unweighted least squares without nonnegativity, b) unweighted nonlinear programming least squares with nonnegativity, c) unweighted least squares with nonnegativity, d) Leersum, e) naive, and f) no rectification.

$$S = \begin{bmatrix} 0.1000 & 0.0010 & 0.0010 & 0.0010 & 0.0010 & 0.0010 \\ 0.0010 & 0.1000 & 0.0010 & 0.0010 & 0.0010 & 0.0010 \\ 0.0010 & 0.0010 & 0.1000 & 0.0010 & 0.0010 & 0.0010 \\ 0.0010 & 0.0010 & 0.0010 & 0.1000 & 0.0010 & 0.0010 \\ 0.0010 & 0.0010 & 0.0010 & 0.0010 & 0.1000 & 0.0010 \\ 0.0010 & 0.0010 & 0.0010 & 0.0010 & 0.0010 & 0.1000 \end{bmatrix} \tag{24}$$

Since the example problem that is used herein is presented with nominally flat surfaces, it is improbable that the diagonal view factors would have such dominant uncertainties. However, our purpose is to compare the impact of the rectification algorithms on the fidelity of the heat transfer calculations. For that purpose, the reader is asked to overlook this impracticality.

Tables 3 and 4 show the mean and standard deviations of the heat fluxes for 1000 trials where the view factors in Eq. (23) were perturbed by values from normal random number generators with standard deviations given by Eq. (24). The rectification schemes are the unweighted and weighted nonnegative least-squares projection methods, Leersum's method, and the naive method. Figure 3 shows histograms for the heat-flux distributions for surface 3.

Table 4 Standard deviations in heat flux for the diagonal-dominated variance case

Surface	Nonnegative least squares Unweighted	Nonnegative least squares Weighted	Leersum	Naive
1	1.7256	0.0468	3.5462	0.1029
2	2.3167	0.0623	5.1518	0.1584
3	3.3123	0.0912	6.4956	0.2284

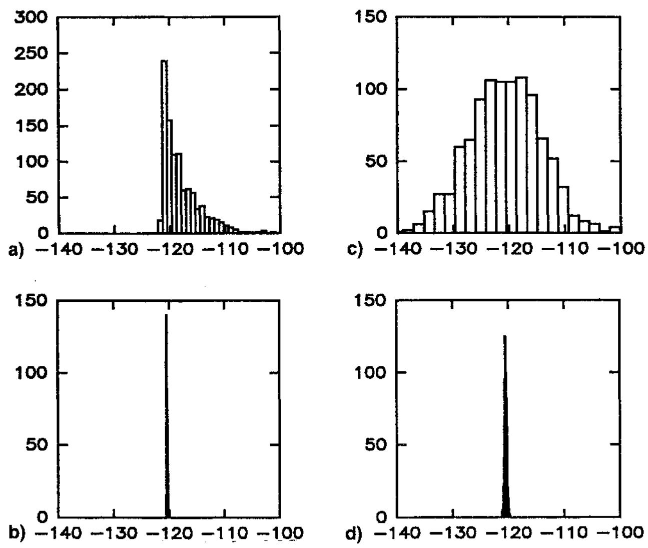

Fig. 3 Histograms of surface-3 heat flux (W/m²) for the diagonal-dominated case: a) unweighted nonnegative least squares, b) weighted nonnegative least squares, c) Leersum, and d) naive.

For this case study, Leersum's rectification is seen to be the least effective at reducing errors in the heat flux. The unweighted nonnegative least-squares projection is about twice as effective as Leersum's scheme, but the weighted nonnegative least-squares projection is an order of magnitude more effective. For this case, the naive rectification is almost as good as the weighted least-squares projection.

Recall that the naive rectification scheme lumps all of the corrections into the diagonal elements for closure enforcement, whereas Leersum's scheme evenly distributes the corrections over all of the nonzero values. Therefore, when the view-factor variance is mostly along the diagonal, we expect the naive scheme to perform well and Leersum's to not perform well. When the variances are all equal Leersum's is expected to perform well, as it did in the previous case study.

Figure 3 shows that there is a considerable skew to the heat flux distributions for the nonnegative least-squares cases. It is believed that this is caused by the nonnegativity constants. The diagonal elements of the view-factor matrix have nominal values that are zero; therefore, the Monte Carlo procedure will produce many negative diagonal view factors that are then set to zero.

Other Unequal Variance

Five additional cases were considered that contained unequal variance: 1) counter-diagonal-dominated, 2) row-dom-

Table 5 Mean heat-flux for the other unequal variance cases

	Nonnegative least squares				
Surface	Unweighted	Weighted	Leersum	Naive	Base
	Counter-diagonal-dominated view-factor variance				
1	−3.5645	−3.6229	−3.4449	−3.1886	−3.6266
2	82.2329	83.9206	83.0833	83.8364	83.9447
3	−119.5026	−120.4466	−119.4577	−119.4717	−120.4772
	Row-dominated view-factor variance				
1	−2.7005	−3.6096	−2.6789	−1.7829	−3.6266
2	82.0741	83.9014	81.9244	79.4481	83.9447
3	−119.0604	−120.4378	−118.8689	−116.4979	−120.4772
	Column-dominated view-factor variance				
1	−2.6462	−3.6033	−3.7737	−3.5184	−3.6266
2	81.8186	83.8880	83.7971	83.8305	83.9447
3	−118.7586	−120.4271	−120.0350	−120.4682	−120.4772
	Upper-triangle-dominated view-factor variance				
1	−3.5754	−3.6222	−3.3351	−3.3087	−3.6266
2	83.6157	83.9198	82.6190	80.2599	83.9447
3	−120.0605	−120.4450	−118.9259	−115.4267	−120.4772
	Random view-factor variance				
1	−3.2880	−3.2593	−3.3021	−2.0925	−3.6266
2	82.4014	82.7955	82.8567	80.3335	83.9447
3	−118.6701	−119.3044	−119.3318	−117.3613	−120.4772

Table 6 Standard deviations in heat flux for the other unequal view-factor variance cases

	Nonnegative least squares			
Surface	Unweighted	Weighted	Leersum	Naive
	Counter-diagonal-dominated view-factor variance			
1	0.5277	0.0368	1.4061	3.4348
2	0.6197	0.0460	2.0133	5.2123
3	0.8957	0.0563	1.6163	6.2030
rms-avg	0.6988	0.0470	1.6973	5.0810
	Row-dominated view-factor variance			
1	1.7794	0.0477	3.6299	5.4372
2	2.7123	0.0630	7.0196	16.1912
3	2.5028	0.0618	5.8428	19.2367
rms-avg	2.3655	0.0579	5.6742	14.8523
	Column-dominated view-factor variance			
1	1.9819	0.0550	3.9047	5.1064
2	3.5176	0.0819	4.8266	5.2548
3	3.4125	0.0701	3.9496	0.3175
rms-avg	3.0521	0.0699	4.2482	4.2343
	Upper-triangle-dominated view-factor variance			
1	0.2454	0.0353	2.0937	11.0725
2	0.3273	0.0438	1.9262	22.9427
3	0.4017	0.0535	2.2979	43.0558
rms-avg	0.3310	0.0448	2.1114	28.8835
	Random view-factor variance			
1	2.6444	2.0461	4.7752	14.7834
2	2.6682	2.0091	6.1736	17.8497
3	2.4439	1.7696	5.3161	15.3694
rms-avg	2.5875	1.9454	5.4521	16.0560

Table 7 Range of naive heat-flux values for the upper-triangle-dominated variance case

Surface	Maximum	Mean	Minimum
1	38.9123	−3.3087	−281.7724
2	106.9031	80.2599	−389.7757
3	991.5615	−115.4267	−205.6421

inated, 3) column-dominated, 4) upper-triangle-dominated, and 5) random variances. The same procedure is followed for these case studies. For all of the cases with regional dominance the base view-factor standard deviation is 0.001 and the value in the dominate region is 0.1. For the counter-diagonal-dominated case the larger values of standard deviation are obviously along the counter diagonal. For the row-dominated and column-dominated cases the larger values are on the second row and second column, respectively. For the upper-triangle-dominated case the six elements in the upper-right corner have the larger values. For the random-variance case the standard deviations were assigned randomly in the range 0–0.1.

Table 5 shows the mean heat flux values, and Table 6 shows the standard deviations of the heat fluxes for the various 1000 trial Monte Carlo studies. The tables reveal that the weighted nonnegative least-squares projection scheme is vastly superior to the others. Overall, its mean heat fluxes most closely agree with the base values, and with the exception of the random-variance case, its standard deviation is one to four orders of magnitude smaller than those for the other schemes. For the random-variance case, the weighted least-squares scheme gives the best results, but the unweighted least-squares and Leersum's schemes also give good results since the uncertainties are more or less evenly distributed.

For some cases, the naive rectification scheme fails completely. Table 7 gives the range of computed heat fluxes for the naive rectification with the upper-triangle-dominated view-factor uncertainties. Clearly, any single heat-flux computation from this set is meaningless.

It should be noted that these are terribly damaged view-factor matrices. For this case, the 95%-confidence uncertainty in view factor is approximately 0.1, or the view factors are considered to have approximately 1-digit accuracy. This would correspond to very crudely computed view factors. However, properly rectified cases yield very meaningful heat-flux computations.

Conclusions

Several view-factor rectification schemes have been compared. Figure 4 summarizes the standard deviation results for

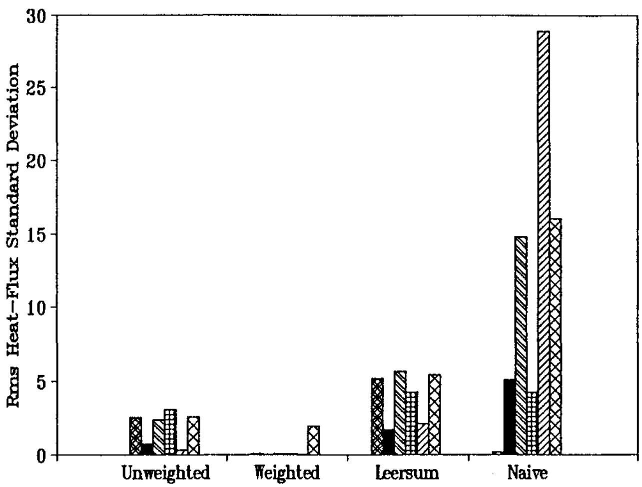

Fig. 4 Summary of rms heat flux standard deviation results for view-factor variance. The bars correspond from left to right for each algorithm to 1) diagonal dominated, 2) counter diagonal, 3) row dominated, 4) column dominated, 5) upper triangle, and 6) random.

heat flux. The naive scheme, where all of the corrections are placed in the diagonal elements of the view-factor matrix, has proven to be erratic and sometimes results in meaningless calculations. Leersum's iterative scheme is also erratic, but on average gives considerably better results than the naive scheme. Leersum's scheme is most viable when the view factors have equal variance. The unweighted version of the nonnegative least-squares projection scheme is better behaved than either the naive or Leersum's scheme. In the cases where the variances were not equally distributed, the weighted nonnegative least-squares projection gives heat-flux results that were orders of magnitude better than the other schemes.

The naive scheme is not recommended. If no knowledge on the relative sizes of the view-factor variances is available, either Leersum's scheme or the unweighted nonnegative least-squares projection will take fairly crudely calculated view factors and compute meaningful heat transfer results. The least-squares projection is recommended since the computational tasks are roughly equivalent and it is about twice as effective. If information is available on the relative variance of the view factors (which is always the case for Monte Carlo integrations), the weighted nonnegative least-squares projection should be used. The weighted nonnegative least-squares projection can be thought of as a numerical filter for noisy view-factor data. In the examples given here, very good heat transfer calculations were made for cases with very crudely defined view-factor data (roughly 1-digit accuracy). View-factor calculations are the most computationally intensive part of many radiation enclosure problems. There is the possibility of considerable improvement in computational efficiency by combining this excellent filter with relatively crude computations of the view-factor values. To properly make such a compromise sensitivity estimates[1] of the heat transfer calculations would be required.

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